

# The Case of the Two Runners

This brain teaser draws on the distinction between scalar and vector quantities ([Outcome](#), [Outcome](#)) and the definitions of displacement, velocity and acceleration ([Outcome](#), [Outcome](#)). It ties directly to the kinematics unit's focus on describing motion using these four quantities ([Topic](#)) and echoes the STS connection between kinematics and everyday activities such as timing a run ([Outcome](#)).

## The puzzle

Two runners, Priya and Jonah, start at the same point on a straight 400 m track and run in opposite directions along the track for 20 s.

Priya runs 60 m east, then turns around and runs 20 m back west, arriving at her final position after the full 20 s.

Jonah runs 40 m west in a straight line, without changing direction, for the same 20 s.

At the end of the 20 s, a judge announces that "one runner has a greater average speed, but the other runner has a greater magnitude of average velocity."

Using the definitions of distance, displacement, average speed and average velocity, determine:

1. Which runner has the greater average speed, and what is that speed?
2. Which runner has the greater magnitude of average velocity, and what is that velocity?
3. Explain, in terms of the definitions of scalar and vector quantities, why the two rankings do not agree.

Assume east is positive and west is negative. Give all answers to two significant figures.

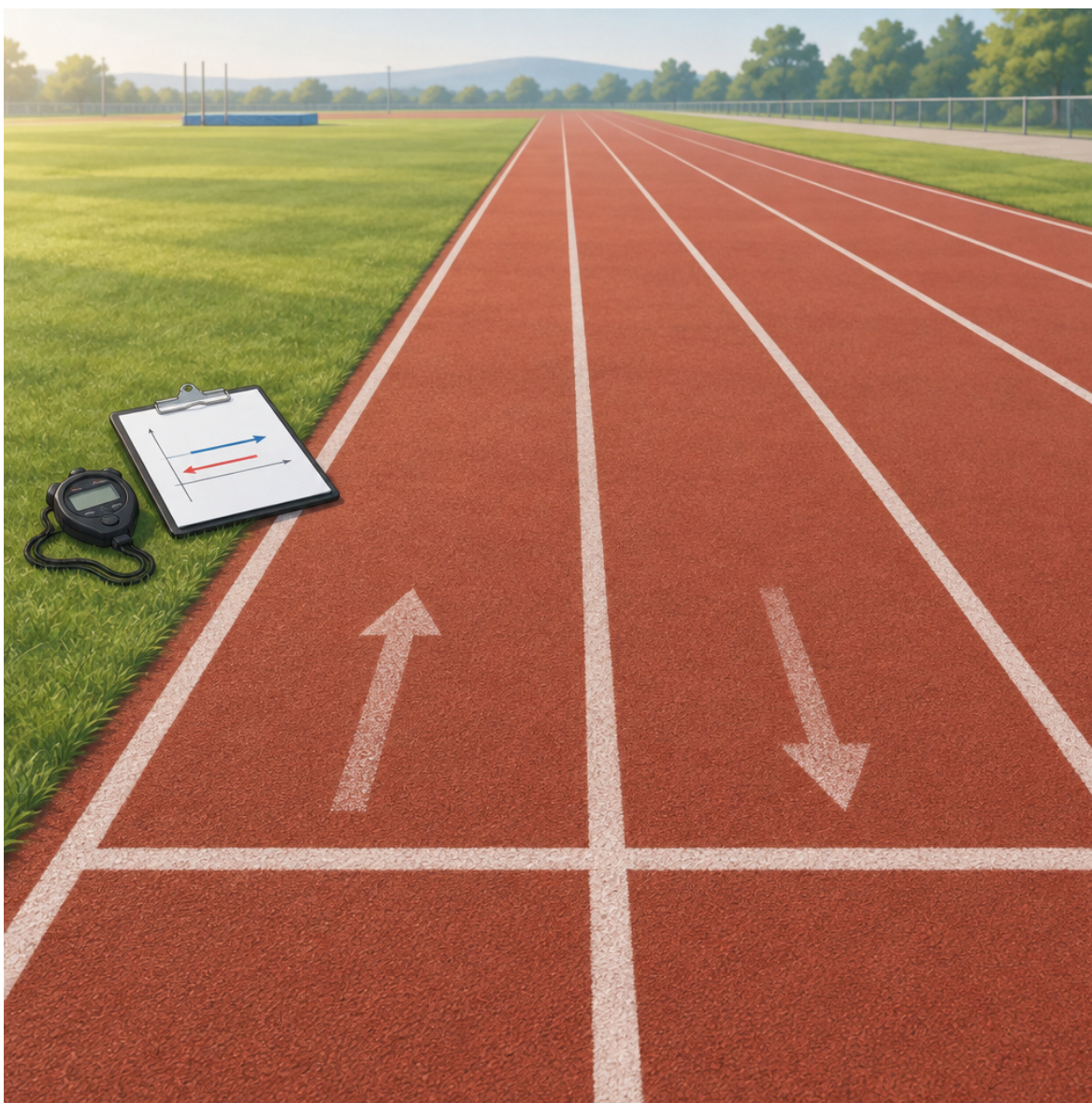


Figure 1: A straight running track viewed from above, painted lane lines stretching into the distance, with two faint directional arrows chalked onto

## Hints

1. Average speed depends only on the total path length travelled, while average velocity depends only on the straight-line change in position from start to finish. Start by finding Priya's total distance travelled versus her net displacement.
2. For Priya: total distance =  $60\text{ m} + 20\text{ m} = 80\text{ m}$ , but her displacement is only  $60\text{ m east} - 20\text{ m west}$ . For Jonah: since he never changes direction, his distance and displacement have the same magnitude.
3. Divide each runner's total distance by  $20\text{ s}$  to get average speed, and divide each runner's net displacement by  $20\text{ s}$  to get average velocity. Compare the two runners on each measure separately before deciding who "wins" which category.

## Answer key

**Priya:**

Total distance = 60 m + 20 m = 80 m

Average speed =  $80 \text{ m} \div 20 \text{ s} = 4.0 \text{ m/s}$

Net displacement = +60 m + (" 20 m) = +40 m (40 m east)

Average velocity =  $40 \text{ m} \div 20 \text{ s} = 2.0 \text{ m/s east}$

**Jonah:**

Total distance = 40 m

Average speed =  $40 \text{ m} \div 20 \text{ s} = 2.0 \text{ m/s}$

Net displacement = " 40 m (40 m west)

Average velocity =  $40 \text{ m} \div 20 \text{ s} = 2.0 \text{ m/s west}$

**Comparison:**

1. Priya has the greater average speed: 4.0 m/s versus Jonah's 2.0 m/s.
2. The magnitudes of their average velocities are equal at 2.0 m/s, so strictly neither runner has a "greater" magnitude of average velocity than the other. The riddle's claim that "the other runner" has a greater magnitude is a trick: the magnitudes tie, even though the directions differ (Priya moves east, Jonah moves west).
3. Average speed is a scalar, calculated from total distance travelled regardless of direction, so backtracking adds to the total ([Outcome](#), [Outcome](#)). Average velocity is a vector, calculated from the net change in position, so Priya's backtracking partially cancels her eastward progress and lowers her velocity relative to her speed ([Outcome](#), [Outcome](#)). This is exactly why the two quantities can rank runners differently: distance accumulates with every step, while displacement only reflects the straight-line result from start to finish.