

Integers and Fractions: Making Sense of Signs and Reciprocals

Sign rules, reciprocals, and order of operations in real-world contexts

Audience: Grade 7 mathematics students, ages 12 to 13, in an Alberta classroom studying the Number strand.

Grades: Grade 7

Subjects: Mathematics

Time: about 60 minutes

Overview

This lesson connects two big ideas from the Grade 7 Number strand: operating with positive and negative numbers, and interpreting multiplication or division of positive fractions and positive decimal numbers ([Outcome](#), [Outcome](#)). Students start by revisiting integer addition and subtraction on a number line, then build the sign rules for integer multiplication and division through guided reasoning rather than memorization alone ([Skills](#), [Knowledge](#)). The lesson then shifts to fractions, where students prove that a fraction times its reciprocal equals 1 and use that relationship to divide fractions with real-world contexts such as recipes ([Knowledge](#), [Understanding](#), [Skills](#)).

The lesson closes with a mixed application station where students solve problems that blend integers, fractions, and order of operations, since the same conventional order of operations governs all three number types ([Knowledge](#), [Understanding](#), [Skills](#)). An individual exit ticket gives a quick summative snapshot of where each student stands on both strands before moving further into the unit.

Outcomes covered

- Students apply operations to positive and negative numbers. ([Outcome](#))
Integer operations show up constantly in real situations such as temperature changes, elevation, and financial transactions, and mastering the sign rules is foundational for algebra in Grade 8 and 9.
- Students apply operations to positive and negative numbers, and interpret multiplication and division of positive fractions and of positive decimal numbers. ([Outcome](#)) Division by a fraction is equivalent to multiplication by its reciprocal. ([Knowledge](#))

Materials

- Whiteboard or chart paper with a drawn vertical number line
- Individual student whiteboards and markers, or notebooks
- Printed or projected problem set for the application station
- Half-sheet exit ticket handout
- Calculators (optional, for checking work)
- Sticky notes or index cards for pair work

Sequence of activities

1. Warm-Up: Integer Number Line Challenge (8 min, whole class)

- Post this question: "A submarine is at " 45 m. It rises 20 m, then dives 12 m. What is its new depth?"
- Give students 90 seconds to solve individually on whiteboards or paper.

- Students can support their addition and subtraction of positive and negative numbers by employing standard algorithms or expressing subtraction as related addition. Expect: " $45 + 20 = 37$ m.
- Ask: "Why does adding a positive number to a negative number sometimes still give a negative result?" Draw out that addition does not always make a number bigger, and subtraction does not always make it smaller ([Knowledge](#)).
- Clarify that the minus sign has two jobs: it can mean "negative" or it can mean "subtract." Show $5 - 8$ rewritten as $5 + (-8)$ to make this explicit ([Knowledge](#), [Skills](#), [Understanding](#)).

If students struggle, model on a physical or drawn vertical number line with sea level at 0. Watch for students who read " 45 " as "minus 45 " but cannot say what that means physically. This sets up today's focus: integers and fractions both follow rules for combining positive and negative quantities.

Sources: [Knowledge](#), [Skills](#), [Understanding](#)

2. Direct Instruction: The Sign Rules for Multiplying and Dividing Integers (12 min, whole class)

- Present the three sign rules on the board: positive $\times$ positive = positive, negative $\times$ negative = positive, positive $\times$ negative (or negative $\times$ positive) = negative ([Knowledge](#)).
- Show that a negative number can always be written as its positive value times -1 , e.g., $-7 = -1(7)$. Use this to explain why two negatives make a positive: multiplying by -1 twice flips direction twice, landing back on positive ([Knowledge](#)).
- Work through: $(-6) \times (-4) = 24$, and $15 \div (-3) = -5$. Ask students to justify each sign using the rule, not just the number.
- Pose: "Will $(-2) \times (-3) \times (-4)$ be positive or negative? Why?" Let students discuss in pairs for 60 seconds, then reveal that three negative factors multiply to a negative result. Generalize: count the negative signs, an even count gives positive, an odd count gives negative ([Skills](#), [Understanding](#)).
- Ask students to write the same product, 24, three different ways using positive and negative factors, e.g., 4×6 , $(-4) \times (-6)$, $(-2) \times (-12)$ ([Skills](#), [Understanding](#)).
- Quick check: cold-call students for the sign (not the value) of: $(-9)(3)$, $(-8)(-8)$, $20 \div (-5)$, $(-12) \div (-4)$.

Formative checkpoint: circulate during step 5 and check that students can generate at least two equivalent expressions for the same product. This confirms understanding of [Understanding](#) before moving to fractions.

Sources: [Skills](#), [Understanding](#), [Knowledge](#)

3. Guided Practice: Dividing Fractions with Reciprocals (12 min, whole class)

- Define reciprocal: the multiplicative inverse of a fraction, where a fraction times its reciprocal equals 1 ([Knowledge](#), [Skills](#)).
- Ask students to find the reciprocal of $3/5$, 7 , and $2/9$. Confirm that the reciprocal of a whole number 7 is $1/7$, since $7 = 7/1$.
- Prove as a class that $(3/5) \times (5/3) = 15/15 = 1$, showing why the product of a fraction and its reciprocal is always 1 ([Knowledge](#), [Skills](#)).
- Introduce the rule: $a/b \div c/d = a/b \times d/c = ad/bc$ ([Knowledge](#)). Model with a recipe context: "A recipe needs $3/4$ cup of flour per batch. A cook has 6 cups of flour and can apply division by a fraction to real-world situations such as sharing food. How many batches can you make?" Set up $6 \div 3/4 = 6 \times 4/3 = 24/3 = 8$ batches.
- Connect to [Understanding](#): ask "What does $6 \div 3/4$ actually mean?" Draw out that it asks how many $3/4$ -cup groups fit into 6 cups, which is why the answer, 8, is larger than 6 ([Understanding](#)).
- Have students try independently: $2/3 \div 1/6$ (answer: 4) and $5 \div 5/8$ (answer: 8). Check answers as a class and ask a volunteer to explain why dividing by a proper fraction gives a quotient greater than

the original number ([Knowledge](#), [Skills](#)).

Formative checkpoint: listen for the phrase "how many groups fit in" when students explain their reasoning in step 5. This is the key conceptual marker for [Understanding](#). Some students will want to just "flip and multiply" without understanding why; press at least two students to justify the rule using the reciprocal proof from step 3.

Sources: [Knowledge](#), [Understanding](#), [Skills](#)

4. Application Station: Mixed Integer and Fraction Problem Set (18 min, pairs)

1. Distribute the problem set (or write on the board) with these four problems for pairs to solve together, showing all steps:
 - a) A hiking group starts at 8°C at the summit at dawn. By noon the temperature has risen 15°C , then a storm drops it 6°C . Find the final temperature. (Answer: $8 + 15 - 6 = 17^{\circ}\text{C}$)
 - b) A scuba diver descends in three equal stages to reach 18 m . Represent this as a product of integers and find the depth of each stage. (Answer: $3 \times (6) = 18$, so each stage is 6 m)
 - c) A pizza recipe calls for $\frac{2}{3}$ cup of sauce per pizza. A cook has 4 cups of sauce. How many pizzas can be made? Then find how much sauce is left over if only 5 pizzas are made. (Answer: $4 \div \frac{2}{3} = 6$ pizzas possible; 5 pizzas use $5 \times \frac{2}{3} = \frac{10}{3}$ cups, leaving $4 - \frac{10}{3} = \frac{2}{3}$ cup)
 - d) Evaluate using order of operations: $(3) \times (4 - 7) + \frac{2}{5} \div \frac{1}{5}$. (Answer: $(3) \times (-3) + 2 = -9 + 2 = -7$) ([Knowledge](#), [Understanding](#), [Skills](#))
2. Circulate and prompt pairs who finish early to write their own real-world word problem that requires both an integer operation and a fraction operation, then trade with another pair to solve.
3. Bring the class back together for the final 4 minutes. Cold-call pairs to present their reasoning for problem (d), focusing on why order of operations applies the same way to integers and fractions as it does to whole numbers ([Knowledge](#)).
4. Ask: "Which problem felt hardest, and why?" Use responses to identify any lingering misconceptions before the exit ticket.

Formative checkpoint: use problem (b) to check that students can represent integer division or multiplication situations as an expression, per [Understanding](#) and [Skills](#). Extension pairs can be asked to find a second way to split the 18 m depth into three unequal negative stages that still multiply or add to 18 .

Sources: [Knowledge](#), [Understanding](#), [Skills](#), [Understanding](#), [Skills](#)

5. Exit Ticket: Show What You Know (10 min, individual)

1. Distribute a half-sheet with three questions:
 - a) Calculate: $(7) + (4) - (10)$. Show your steps. (Answer: $7 + 4 - 10 = 1$)
 - b) Calculate: $\frac{3}{4} \div \frac{2}{3}$. Show your steps and simplify. (Answer: $\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$, or $1 \frac{1}{8}$)
 - c) Division can be applied to real-world situations such as sharing food and cutting materials. Write a multiplication expression for this situation and state whether each stage's depth change is positive or negative and why. (Division by a fraction is equivalent to multiplication by its reciprocal)
2. Students work silently and independently for 8 minutes.
3. Collect exit tickets as students leave, or have them place them in a labelled bin.
4. Use a 2-minute scan of results to flag 3 to 5 students needing a follow-up small group tomorrow on either integer signs or fraction division.

This is the summative check for the lesson. Sort responses into three piles while scanning: solid on both integers and fractions, solid on one but not the other, and needs support on both. Use this sort to plan tomorrow's small-group instruction.

Sources: [Outcome](#), [Skills](#), [Skills](#)

Differentiation

Extension

- Ask students to prove why an even number of negative factors always produces a positive product, using the " $a = -1(a)$ " rule from [Knowledge](#) as their starting point.
- Challenge students to design a three-step real-world scenario (temperature, elevation, or money) that requires both integer addition/subtraction and fraction multiplication or division to solve, then trade with a partner.
- Have students investigate what happens when dividing two fractions with common denominators, using the shortcut $a/b \div c/b = a/c$, and explain why it works using the standard reciprocal method ([Knowledge](#)).

Support

- Provide a physical or laminated vertical number line for students to place counters on when adding and subtracting integers.
- Offer a reference card showing the three sign rules for multiplication and division of integers side by side with worked examples ([Knowledge](#)).
- Allow fraction division problems to be solved first by rewriting both fractions with a common denominator and dividing numerators directly, as in $a/b \div c/b = a/c$, before introducing the reciprocal method ([Knowledge](#)).

Inclusive supports

- Pair students strategically so that a student who is confident with integer rules works alongside a student who is confident with fraction rules, so each can teach the other.
- Provide sentence starters such as "I know the answer is negative because..." to support students who find verbal justification difficult.
- Allow use of a calculator to check final answers so that computation is not a barrier to demonstrating conceptual understanding of signs and reciprocals.

Assessment

Formative. During the sign-rules mini-lesson, students generate at least two equivalent multiplication expressions for the same product using positive and negative factors.

Look for: A pair or individual correctly writes expressions such as 4×6 , $(-4) \times (-6)$, and $(-2) \times (-12)$ all equal to 24, showing they understand products can be represented in many equivalent ways. ([Skills](#), [Understanding](#))

Formative. During guided practice, the teacher listens for students explaining fraction division using the phrase or idea of "how many groups fit in" when solving a division-by-a-fraction problem.

Look for: A student correctly explains that $6 \div 3/4$ asks how many $3/4$ -sized groups fit into 6, and that the quotient, 8, is larger than 6 because a proper fraction is being divided into. ([Understanding](#), [Knowledge](#))

Peer. In the application station, pairs trade a self-written word problem that requires integer operations, then solve their partner pair's problem and check the answer together.

Look for: Both pairs agree on a correct final answer and can point to the specific operation rule used, such as the sign rule for integer multiplication or the reciprocal rule for fraction division. ([Skills](#), [Knowledge](#))

Summative. Exit ticket with three questions covering integer addition and subtraction, and representing a real-world integer situation as a multiplication expression.

Look for: Correct answers of " 1, $1\frac{1}{8}$ (or $\frac{9}{8}$), and $5 \times (" 7) = " 35$ with a clear statement that each stage's depth change is negative because the submarine moves downward. ([Skills](#), [Skills](#), [Understanding](#))

Vocabulary

- **Reciprocal:** The multiplicative inverse of a fraction. Multiplying a fraction by its reciprocal always gives 1, e.g., $\frac{3}{5} \times \frac{5}{3} = 1$.
- **Proper fraction:** A fraction in which the numerator is less than the denominator, such as $\frac{2}{3}$ or $\frac{5}{8}$.
- **Order of operations:** The agreed sequence for evaluating a numerical expression: brackets, exponents, multiplication and division (left to right), then addition and subtraction (left to right). This sequence applies the same way to integers, fractions, and decimal numbers.
- **Integer:** A whole number that can be positive, negative, or zero, such as " 7, 0, or 12.
- **Equivalent expression:** A different way of writing the same value or product, such as writing 24 as 4×6 or as $(" 4) \times (" 6)$.

For families

- Ask your student to explain why multiplying two negative numbers gives a positive answer, using a real-life example like temperature change or money owed.
- Practice fraction division at home using a recipe: if a recipe needs $\frac{3}{4}$ cup of an ingredient and you want to know how many batches you can make from a full container, that is a division-by-a-fraction problem.
- Encourage your student to check their homework answers by asking whether the sign and size of the answer make sense for the situation, not just whether the calculation was performed correctly.

Sources

- [Outcome](#): "Students interpret multiplication and division of positive fractions and of positive decimal numbers."
- [Outcome](#): "Students apply operations to positive and negative numbers."
- [Knowledge](#): "Division by a fraction is equivalent to multiplication by its reciprocal"
- [Understanding](#): "The quotient of any quantity and a fraction can be interpreted as the number of fraction-sized groups that compose the quantity."
- [Skills](#): "Relate a number to its reciprocal. Prove that multiplication of a fraction and its reciprocal is 1."
- [Understanding](#): "Equivalent expressions can facilitate multiplication and division of decimal numbers."
- [Knowledge](#): "Multiplication and division of a decimal number by a decimal number can be supported by processes"
- [Skills](#): "Multiply and divide decimal numbers, including dividing a natural number by a decimal number."
- [Skills](#): "Model multiplication of fractions. Relate the product of a fraction by a fraction to part of a part."
- [Knowledge](#): "The product of two fractions is the fraction resulting from multiplication of the numerators and multiplication of the denominators"
- [Understanding](#): "The product of two fractions can be interpreted as part of a part."

- [Knowledge](#): "The conventional order of operations applies to integers, fractions, and decimal numbers."
- [Understanding](#): "Equivalent forms of mathematical expressions can facilitate evaluation."
- [Skills](#): "Evaluate numerical expressions according to the order of operations."
- [Knowledge](#): "Addition does not always result in a greater number, and subtraction does not always result in a smaller number."
- [Skills](#): "Add and subtract any two integers."
- [Understanding](#): "Addition and subtraction of integers can be represented as numerical expressions."
- [Skills](#): "Multiply and divide any two integers. Investigate whether a product or quotient of three or more integers will be positive or negative."
- [Understanding](#): "Products and quotients can be represented as numerical expressions in infinitely many ways."
- [Knowledge](#): "The product or quotient of two positive numbers is positive, two negative numbers is positive, a negative number and a positive number is negative"

All content, contexts, and imagery are age-appropriate for Grade 7 students and free of culturally sensitive or exclusionary material.