

Analyzing Quadratic Functions: Standard Form, Vertex Form, and Key Features

Connecting standard form and vertex form through completing the square

Audience: Grade 11 students enrolled in Mathematics 20-1, preparing for further study of quadratic equations, inequalities, and systems, and ultimately for diploma-level assessment.

Grades: Grade 11

Subjects: Mathematics

Time: about 60 minutes

Overview

This lesson develops student ability to analyze quadratic functions given in standard form, $y = ax^2 + bx + c$, by converting them to vertex form, $y = a(x - p)^2 + q$, through completing the square ([Outcome](#), [Outcome](#)). Students practice extracting the vertex, domain, range, direction of opening, axis of symmetry, and x- and y-intercepts from both forms, drawing on factoring skills ([Outcome](#)) and equation-solving strategies ([Outcome](#)). The lesson closes with an applied modelling problem involving a parabolic arch bridge, situating the algebra in a real-world context consistent with the applications described in [Resource](#) and [Resource](#).

The lesson sequence moves from direct instruction with two fully worked examples, through paired analysis practice with a self-marking answer key, to a small-group modelling task, and finishes with an individual exit ticket that doubles as a formative check for the following lesson on quadratic equations and inequalities ([Outcome](#), [Outcome](#)).

Outcomes covered

- Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts. ([Outcome](#))
Vertex form makes the vertex, axis of symmetry, and direction of opening immediately visible, providing an efficient way to graph and interpret quadratic functions once students can convert to this form.
- Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts, and to solve problems. ([Outcome](#))
Students must be able to analyze quadratic functions of the form $y = ax^2 + bx + c$ to extract graphical characteristics.
- Factor polynomial expressions of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$, and related forms, where a, b and c are rational numbers. ([Outcome](#))
Factoring is one method used to find x-intercepts of a quadratic function and is reinforced during the guided practice and application activities.
- Solve problems that involve quadratic equations. ([Outcome](#))
Finding x-intercepts of a quadratic function requires solving the associated quadratic equation, connecting graphical analysis to algebraic problem solving.

Materials

- Whiteboard and markers
- Projector or document camera
- Handout with four practice quadratic functions (guided practice activity)

- Parabolic arch bridge problem handout
- Exit ticket handout
- Scientific or graphing calculators
- Answer key for guided practice (teacher copy)

Sequence of activities

1. Activation: From $y = a(x - p)^2 + q$ to Vertex Recognition (8 min, whole class)

1. Display three functions on the board: $y = 2(x - 3)^2 + 1$, $y = -(x + 2)^2 - 4$, $y = 0.5(x - 0)^2 + 5$.
2. Ask students, working individually for 2 minutes, to identify the vertex, direction of opening, and axis of symmetry for each without graphing.
3. Cold-call three students to share one function each. Confirm: vertex (p, q) , opens up if $a > 0$ and down if $a < 0$, axis of symmetry $x = p$.
4. State the learning intention: 'Today we connect the vertex form and the standard form of a quadratic function, and we use both to determine key features of the graph.'
5. State the success criteria: 'I can convert $y = ax^2 + bx + c$ to vertex form by completing the square, and I can state the vertex, domain, range, direction of opening, axis of symmetry, and intercepts from either form.'

This reviews [Outcome](#) before introducing [Outcome](#). If students struggle with sign of q or direction for negative a , revisit the transformation language: horizontal shift right p , vertical shift up q , vertical stretch/reflection by a .

Sources: [Outcome](#)

2. Direct Instruction: Completing the Square to Reach Vertex Form (15 min, whole class)

1. Present the standard form $y = ax^2 + bx + c$ and explain that every quadratic function can be rewritten in vertex form $y = a(x - p)^2 + q$ by completing the square.
2. Work through Example 1 on the board: convert $y = x^2 - 6x + 5$ to vertex form.
 - Group the x -terms: $y = (x^2 - 6x) + 5$.
 - Take half of -6 , square it: $(-3)^2 = 9$. Add and subtract 9 inside: $y = (x^2 - 6x + 9 - 9) + 5$.
 - Rewrite as a perfect square: $y = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$.
 - State the vertex $(3, -4)$, axis of symmetry $x = 3$, opens up since $a = 1 > 0$.
3. Work through Example 2 with a leading coefficient other than 1: $y = 2x^2 + 8x + 3$.
 - Factor 2 from the x -terms: $y = 2(x^2 + 4x) + 3$.
 - Complete the square inside brackets: half of 4 is 2, squared is 4. $y = 2(x^2 + 4x + 4 - 4) + 3$.
 - Distribute the subtracted 4 by the factored 2: $y = 2(x + 2)^2 - 8 + 3 = 2(x + 2)^2 - 5$.
 - State vertex $(-2, -5)$, axis of symmetry $x = -2$, opens up.
4. Show how to find the y -intercept directly from standard form (set $x = 0$, $y = c$) and the x -intercepts by factoring or the quadratic formula when the trinomial does not factor nicely, connecting to [Outcome](#) and [Outcome](#).
5. For Example 2, find x -intercepts using the quadratic formula: $x = (-8 \pm \sqrt{64 - 24}) / 4 = (-8 \pm \sqrt{40}) / 4$, giving approximate values $x \approx -0.42$ and $x \approx -3.58$.
6. Summarize domain (all real numbers, $x \in \mathbb{R}$, for any quadratic) and range ($y \geq q$ if $a > 0$, $y \leq q$ if $a < 0$), tying back to the vertex found in each example.

This activity directly builds [Outcome](#) skills using the completing-the-square technique referenced in [Resource](#) and [Resource](#). Watch for the common error of forgetting to distribute the factored coefficient onto the subtracted term in step 3. Pause after each example for a thumbs-up/thumbs-down check.

Sources: [Outcome](#), [Outcome](#), [Outcome](#), [Outcome](#), [Resource](#), [Resource](#)

3. Guided Practice: Partner Analysis of Quadratic Functions (15 min, pairs)

- Distribute the following four functions, one per pair to start, then rotate:
 - $y = x^2 + 4x - 5$
 - $y = -2x^2 + 12x - 7$
 - $y = 3(x - 1)^2 + 2$
 - $y = x^2 - 2x - 8$
- For each function, partners determine: vertex, axis of symmetry, direction of opening, domain, range, y-intercept, and x-intercepts (by factoring where possible).
- Partners record answers on a shared answer sheet with columns for each characteristic.
- After 10 minutes, project the answer key and have pairs self-mark, flagging any function where their vertex or intercepts disagree with the key.
- Circulate throughout and prompt pairs stuck on factoring by asking, 'What two numbers multiply to give the constant term and add to give the middle coefficient?'

Answer key: a) vertex $(-2, -9)$, axis $x = -2$, opens up, D: $x \in \mathbb{R}$, R: $y \geq -9$, y-int -5 , x-ints $x = 1$ and $x = -5$ (factors $(x+5)(x-1)$). b) vertex $(3, 11)$, axis $x = 3$, opens down, D: $x \in \mathbb{R}$, R: $y \leq 11$, y-int -7 , x-ints found via quadratic formula $x = (12 \pm \sqrt{88})/4$, i.e. $x \approx 0.65$ and $x \approx 5.35$. c) vertex $(1, 2)$, axis $x = 1$, opens up, D: $x \in \mathbb{R}$, R: $y \geq 2$, y-int 5 , no real x-intercepts since minimum value 2 is above the x-axis. d) vertex $(1, -9)$, axis $x = 1$, opens up, D: $x \in \mathbb{R}$, R: $y \geq -9$, y-int -8 , x-ints $x = 4$ and $x = -2$ (factors $(x-4)(x+2)$). This addresses [Outcome](#), [Outcome](#), [Outcome](#), and [Outcome](#), and mirrors the practice style in [Resource](#).

Sources: [Outcome](#), [Outcome](#), [Outcome](#), [Outcome](#), [Resource](#)

4. Application Problem: Modelling a Parabolic Arch (15 min, small group)

- Present the problem: 'An arch bridge support can be modelled by $h(x) = -0.05x^2 + 2x$, where h is the height in metres above the roadway and x is the horizontal distance in metres from the left base of the arch.'
- In groups of three or four, students determine:
 - The vertex of the arch (the maximum height and where it occurs).
 - The domain that makes physical sense for this bridge (where the arch is above the roadway, $h(x) \geq 0$).
 - The horizontal distance the arch spans (the x-intercepts).
- Guide groups to complete the square: $h(x) = -0.05(x^2 - 40x) = -0.05(x^2 - 40x + 400 - 400) = -0.05(x - 20)^2 + 20$. Vertex is $(20, 20)$, so the arch reaches a maximum height of 20 m at $x = 20$ m.
- Find x-intercepts by factoring or setting $h(x) = 0$: $-0.05x(x - 40) = 0$, giving $x = 0$ and $x = 40$. The arch spans 40 m.
- State the physically sensible domain, $0 \leq x \leq 40$, and range, $0 \leq h(x) \leq 20$, distinguishing this from the full mathematical domain and range of the function.
- Groups share one finding each with the class during a 3-minute whole-class debrief, and the teacher records the maximum height, span, and restricted domain/range on the board.

This connects [Outcome](#), [Outcome](#), and [Outcome](#) to a real-world modelling context, aligned with the applications described in [Resource](#) and [Resource](#) (parabolic paths, architecture and design). Emphasize that the mathematical domain and range ($x \in \mathbb{R}$, $h \geq 20$) differ from the applied domain and range restricted by the physical situation.

Sources: [Outcome](#), [Outcome](#), [Outcome](#), [Resource](#), [Resource](#)

5. Closure: Exit Ticket and Preview of Next Steps (7 min, individual)

1. Distribute an exit ticket with one function: $y = x^2 + 6x - 5$.
2. Students individually determine the vertex, axis of symmetry, direction of opening, domain, range, and y-intercept, showing the completing-the-square steps.
3. Students submit the exit ticket before leaving.
4. Preview the next lesson: solving quadratic equations algebraically (factoring, quadratic formula) and quadratic inequalities, connecting to [Outcome](#), [Outcome](#), and [Outcome](#).

Exit ticket answer: $y = x^2 + 6x - 5 = (x^2 + 6x) - 5 = (x^2 + 6x + 9 - 9) - 5 = (x + 3)^2 + 9 - 5 = (x + 3)^2 + 4$. V: (3, 4), axis $x = -3$, opens up, D: $x \in \mathbb{R}$, R: $y \geq 4$, y-int: -5. Use results to group students for tomorrow's factoring review ([Outcome](#)) versus quadratic formula practice ([Outcome](#)).

Sources: [Outcome](#), [Outcome](#), [Outcome](#), [Outcome](#), [Outcome](#)

Differentiation

Extension

- Challenge students to derive the general vertex formula $p = -b/(2a)$, $q = c - b^2/(4a)$ by completing the square on $y = ax^2 + bx + c$ symbolically, then verify it against Example 2 from the direct instruction activity.
- Ask students to model a second real-world parabolic scenario (for example, a basketball free-throw trajectory or a satellite dish cross-section) by researching or estimating realistic coefficients and finding the maximum height and horizontal span.
- Have students explore how changing the sign and magnitude of a in $y = a(x - p)^2 + q$ affects the range and applied domain of the arch bridge model in the application activity, then generalize a rule.

Support

- Provide a completed reference card showing the completing-the-square steps side by side for a simple example ($a = 1$) and a more complex example ($a \neq 1$), for students to use during guided practice.
- Reduce the guided practice set to two functions (one with $a = 1$, one already in vertex form) and allow use of a graphing calculator to verify the vertex and intercepts visually.
- Pre-fill the first two lines of the arch bridge completing-the-square work in the application activity so groups can focus on interpreting the vertex and intercepts rather than the algebraic manipulation.

Inclusive supports

- Use an exit ticket to assess student understanding of quadratic function characteristics.
- Use consistent color-coding on the board (one color for the vertex, one for the axis of symmetry, one for intercepts) to support students who benefit from visual anchors.
- Allow pairs to be strategically grouped so that a student who has recently mastered factoring can support a peer during the guided practice rotation.

Assessment

Formative. Thumbs-up/thumbs-down checks after each worked example during direct instruction, and circulation during paired practice to observe completing-the-square technique and identification of vertex, intercepts, domain, and range.

Look for: Students correctly state the vertex, axis of symmetry, and direction of opening immediately after each worked example; pairs correctly complete at least three of the four practice functions with

matching answers on the shared answer key. ([Outcome](#), [Outcome](#))

Formative. Small-group debrief on the parabolic arch modelling problem, where each group reports one finding (maximum height, span, or restricted domain/range).

Look for: Groups correctly identify the vertex (20, 20), the x-intercepts (0 and 40), and articulate why the applied domain and range differ from the full mathematical domain and range of the quadratic function. ([Outcome](#), [Outcome](#), [Outcome](#))

Formative. Exit ticket requiring students to convert $y = x^2 + 6x - 5$ to vertex form by completing the square and to state vertex, axis of symmetry, direction of opening, domain, range, and y-intercept.

Look for: Correct vertex form $(x - 3)^2 + 4$, vertex (3, 4), axis $x = 3$, opens down, domain $x \in \mathbb{R}$, range $y \leq 4$, y-intercept -5 , with completing-the-square steps shown. ([Outcome](#), [Outcome](#))

Vocabulary

- **Vertex form:** The form $y = a(x - p)^2 + q$, where (p, q) is the vertex of the parabola, a determines the direction of opening and vertical stretch, and $x = p$ is the axis of symmetry.
- **Standard form:** The form $y = ax^2 + bx + c$, where $a \neq 0$, and b and c are real numbers, and the y-intercept can be read directly as c.
- **Completing the square:** An algebraic technique used to rewrite a quadratic expression from standard form into vertex form by creating a perfect square trinomial.
- **Axis of symmetry:** The vertical line $x = p$ that passes through the vertex of a parabola, dividing the graph into two mirror-image halves.
- **Domain and range:** The domain of any quadratic function is all real numbers, $x \in \mathbb{R}$. The range depends on the direction of opening and the y-coordinate of the vertex: $y \geq q$ when $a > 0$, or $y \leq q$ when $a < 0$.

Sources

- [Outcome](#): "Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the: vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts."
- [Outcome](#): "Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems."
- [Outcome](#): "Solve problems that involve quadratic equations."
- [Outcome](#): "Solve problems that involve quadratic inequalities in one variable."
- [Outcome](#): "Factor polynomial expressions of the form: where a, b and c are rational numbers."
- [Resource](#): "This assignment booklet includes tasks on graphing and analyzing quadratic functions, converting forms, completing the square, and modelling real-world scenarios using vertex and standard forms."
- [Resource](#): "This module explains vertex and standard forms, graphing techniques, completing the square, and modelling parabolic paths, such as trajectories and structures."
- [Resource](#): "This module explains graphing, factoring, completing the square, and the quadratic formula, and shows real-world applications in architecture and design."

All content involves standard algebraic manipulation and a bridge engineering context with no sensitive material, appropriate for Grade 11 Mathematics 20-1 students.