

The Vanishing Vertex: A Quadratic Function Puzzle Challenge

This puzzle asks you to reconstruct a full quadratic function from partial graph information, the same analytical work required when identifying vertex, intercepts, direction of opening and axis of symmetry from a quadratic function ([Outcome](#), [Outcome](#), [Outcome](#)). Recovering the equation from its roots draws on factoring polynomial expressions ([Outcome](#)) and on solving problems that involve quadratic equations ([Outcome](#)).



Figure 1: A sheet of graph paper resting on a wooden desk beside a mechanical pencil, a clear plastic ruler and a scientific calculator, with a faint

The puzzle

A parabola crosses the x -axis at $x = 1$ and $x = 5$. It opens downward, and its y -intercept is $(0, 10)$.

Without graphing, determine:

1. The coordinates of the vertex.
2. The equation of the parabola written in the form $y = a(x - p)^2 + q$.
3. The maximum value of the function.

Hints

1. The axis of symmetry of a parabola always lies exactly halfway between its two x-intercepts.
2. Write the function in factored form, $y = a(x - 1)(x - 5)$, then use the given y-intercept to solve for a.
3. The axis of symmetry is $x = 3$. Substitute $x = 0$ into $y = a(x - 1)(x - 5)$ and set the result equal to -10 to find a. Then substitute $x = 3$ into the factored equation to find the vertex's y-value.

Answer key

Vertex: (3, 8)

Equation in vertex form: $y = -2(x - 3)^2 + 8$

Maximum value: 8

Reasoning:

The x-intercepts, $x = 1$ and $x = 5$, give the factored form $y = a(x - 1)(x - 5)$, an application of factoring polynomial expressions ([Outcome](#)). The axis of symmetry sits halfway between the roots, at $x = (1 + 5)/2 = 3$.

Substituting the y-intercept (0, -10) into the factored form solves for a:

$$-10 = a(0 - 1)(0 - 5)$$

$$-10 = a(-1)(-5)$$

$$-10 = 5a$$

$$a = -2$$

The negative value of a confirms the parabola opens downward, consistent with the given description and with the direction of opening identified when analyzing quadratic functions ([Outcome](#), [Outcome](#)).

The full factored equation is $y = -2(x - 1)(x - 5)$.

To find the vertex, substitute $x = 3$ (the axis of symmetry) into this equation:

$$y = -2(3 - 1)(3 - 5)$$

$$y = -2(2)(-2)$$

$$y = 8$$

The vertex is (3, 8), and since a is negative, this point is a maximum. The maximum value of the function is 8, occurring at $x = 3$.

Writing the equation in vertex form, $y = a(x - p)^2 + q$, using vertex (3, 8) and $a = -2$, gives:

$$y = -2(x - 3)^2 + 8$$

This matches the structure required when analyzing quadratic functions of the form $y = a(x - p)^2 + q$ to determine vertex, direction of opening and axis of symmetry ([Outcome](#)).